$$W = \int_i^f F\,dx = \int_i^f \frac{dp}{dt}\,dx = \int_i^f m\frac{d}{dt}(\gamma u)\,dx$$

$$p = \gamma \underline{m}\, u$$

$$p = mu \quad \text{(classical)}$$

$$= m\int_i^f \frac{dx}{dt}\, d(\gamma u) = m\int_i^f u\,(\gamma\,du + u\,d\gamma)$$

C.O.V. $x = \gamma u$

$$= m\int_i^f u\gamma\,\underline{\underline{du}} + u^2 d\gamma = m\int_i^f \cancel{u\gamma}\frac{c^2}{\cancel{u}\gamma^{\cancel{3}\,2}}\,d\gamma + u^2 d\gamma$$

$$\boxed{\begin{aligned}\frac{d\gamma}{du} &= \frac{d}{du}\frac{1}{\sqrt{1-\frac{u^2}{c^2}}} = -\frac{1}{2}\frac{1}{\sqrt{1-\frac{u^2}{c^2}}^{\,3}}\left(\frac{-2u}{c^2}\right)\\ &= \gamma^3\frac{u}{c^2} \;//\end{aligned}}$$

$$= m\int_i^f \left(\frac{c^2}{\gamma^2} + u^2\right) d\gamma$$

$$= m\int_i^f \left[c^2\left(1 - \cancel{\frac{u^2}{c^2}}\right) + \cancel{u^2}\right] d\gamma = mc^2\int_i^f d\gamma$$

$$= mc^2\,(\gamma_f - \gamma_i)$$

$$\boxed{u_i = 0 \rightarrow u_f = u}$$

$$K = mc^2(\gamma - 1)$$

$$\underbrace{K + mc^2}_{E} = \gamma mc^2$$

$$E = \gamma mc^2$$

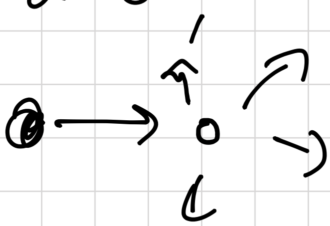
if $u_f = u$ is small compared to c, $u/c \ll 1$,

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}} \approx 1 + \frac{u^2}{2c^2}$$

$$E \approx mc^2 + \frac{u^2 m \cancel{c^2}}{2\cancel{c^2}}$$

$$= mc^2 + \frac{mu^2}{2}$$

if $u = 0$, $E = mc^2$ because $\gamma = 1$



$$E_f - E_i = c^2 \Delta m$$

The energy - momentum relation

$$\boxed{K = \frac{1}{2} m v^2 = \frac{p^2}{2m}}$$

$$p = m\gamma u \qquad E = mc^2\gamma$$

$$\gamma = \frac{1}{\sqrt{1-u^2/c^2}}$$

$$p^2 = \frac{m^2u^2}{1-\frac{u^2}{c^2}} = m^2\left[\frac{1\times c^2}{1-u^2/c^2} + \frac{u^2-1\times c^2}{1-u^2/c^2}\right]$$

$$= \frac{m^2c^2}{1-u^2/c^2} + m^2c^2\underbrace{\frac{u^2-c^2}{c^2-u^2}}_{-1}$$

$$= \frac{1}{c^2}\frac{m^2c^4}{1-u^2/c^2} - m^2c^2$$

$$= \frac{1}{c^2}E^2 - m^2c^2$$

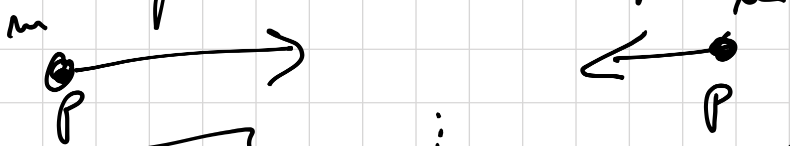
$$c^2p^2 + m^2c^4 = E^2$$

$$E = \sqrt{c^2p^2 + m^2c^4}$$

Center of momentum frame

Laboratory frame

Center of momentum frame:



$E=\sqrt{p^2c^2+m^2c^4}$ $\qquad$ $E=\sqrt{p^2c^2+m^2c^4}$

$$Mc^2 = E = 2\sqrt{p^2c^2+m^2c^4}$$

$$M = \frac{2}{c^2}\sqrt{p^2c^2+m^2c^4}$$

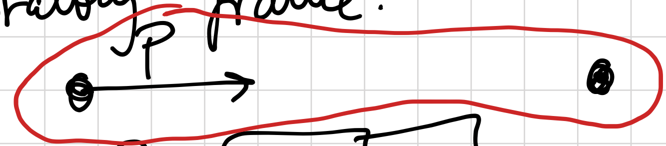
$$= \frac{2pc}{c^2}\sqrt{1+\frac{m^2c^4}{p^2c^2}}$$

$$= \frac{2p}{c}\sqrt{1+\ldots\cancel{x}}$$

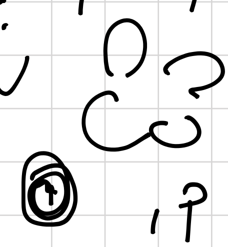
$$\approx \frac{2p}{c}$$

$$P = p - p = 0$$

Laboratory frame:



$p,\ E=\sqrt{m^2c^4+p^2c^2}$ $\quad$ $p=0,\ E=mc^2$



$, P, E_M$

$$E_\mu = \sqrt{m^2c^4 + p^2c^2} + mc^2$$

$$p_\mu = p$$

$$E_\mu = \sqrt{M^2c^4 + p^2c^2}$$

$$E_\mu^2 - p^2c^2 = M^2c^4 = m^2c^4 + m^2c^4 + \cancel{p^2c^2}$$

$$+ 2\sqrt{m^2c^4 + p^2c^2}\, mc^2$$

$$- \cancel{p^2c^2}$$

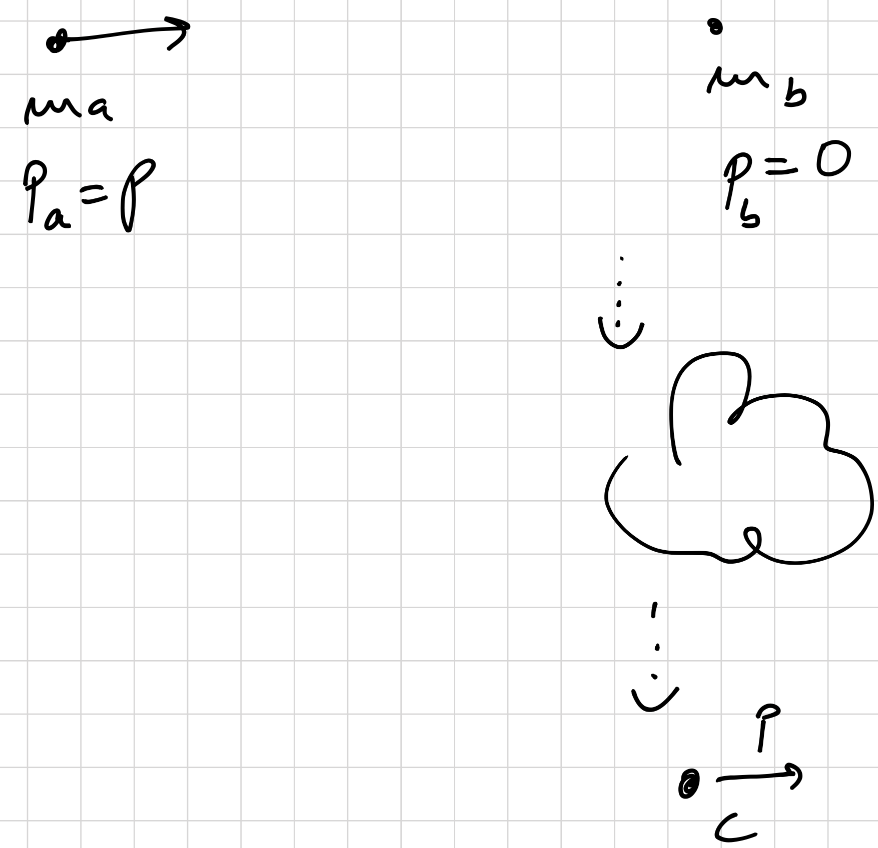
$$M^2c^4 = 2m^2c^4 + 2\sqrt{m^2c^4 + p^2c^2}\, mc^2$$

$$M^2c^2 = 2m^2c^2 + 2m\, pc\sqrt{1 + \frac{m^2c^2}{p^2}}$$

$$M^2 = \underline{2m^2} + 2m\frac{p}{c}\sqrt{1 + X}$$

$$M \approx \sqrt{2m\frac{p}{c}} = \sqrt{2m}\sqrt{\frac{p}{c}}$$

Special session



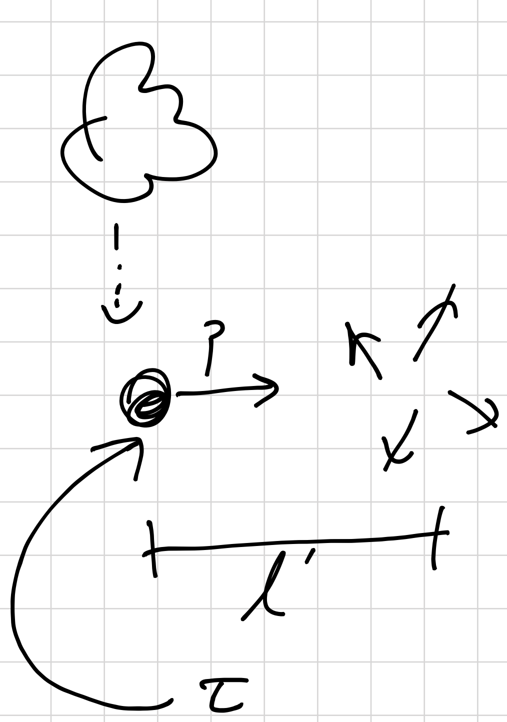
1) Calculate the rest mass of C.

$$E = \sqrt{m_A^2c^4+p^2c^2} + m_B c^2 = \underline{\underline{\sqrt{m_c^2c^4+p^2c^2}}}$$

$$m_c^2c^4+\cancel{p^2c^2} = m_A^2c^4+\cancel{p^2c^2}+m_B^2c^4$$
$$+ 2m_B c^2\sqrt{m_A^2c^4+p^2c^2} \quad \Big| : c^4$$

$$\underline{\underline{m_c^2}} = m_A^2 + m_B^2 + \underbrace{2\frac{m_B}{c^2}\sqrt{m_A^2c^4+p^2c^2}}$$

2)



$$\tau\gamma = t' = \frac{l'}{v}$$

$$\tau = \frac{l'}{\gamma v} \leftarrow \; = \frac{l' m_c}{p}$$

$$p = m_c \gamma v$$